

## Credit Arbitrage Model of Swaps

Zhang Rengui, Xia Qiuhui

Department of Economics, Songtian College of Guangzhou University, Guangzhou, P.R. China, 511370  
(E-mail: mathsren@foxmail.com, lanzhou\_xqh@163.com)

**Abstract** Swap market is one of most rapidly growing financial product markets. However, the profit allocation system of swaps in the international world is far from sound and the flow of arbitrage with swaps has not been theorized. Therefore, this paper has designed a sound program to allocate the profit between the parties to the swap, and has proposed a credit arbitrage model of swaps, which includes detailed process of arbitrage, sound profit allocating program, law of cost and forms of swaps. This paper has elaborated the model in the form of calculation. The clear steps and simple calculation method make it easy to seek solutions through equations. Finally, this paper has, through case-study, proved the practicability of the model and demonstrated that the company with a high credit rating can gain corresponding compensation by virtue of its credit advantage.

**Key words** Financial swaps; Interest rate swaps; Credit arbitrage model of swaps; Theory of comparative advantage

### 1 Introduction

Financial swaps used to be well-known as the most important financial innovation in the western financial world since 1980s. It has been developing very rapidly since it came into existence in 1982. Nowadays, a great number of large-scale multinational banks and investment banking institutions provide swap transaction services. The biggest swap transaction markets lie in London and New York. By the end of 1992, the total outstanding debts of swap markets had amounted up to £ 665,000,000,000. China began to pilot interest rate swap transactions in the year of 2006 when Guangdong Development Bank and China Everbright Bank closed their first swap transaction. In August, 2007, Bank of China released a report saying that from now on it would actively follow the reform process of interest rate and exchange rate systems, actively and prudently promote the development of derivative market, comprehensively carry out RMB interest rate swap transactions, and study and launch derivative products such as forward rate agreement along with currency swaps between RMB and FX as well as credit derivatives. With the boom of swap market, the problem of allocating profit between parties to a swap has been pushed to spotlight, since the time and energy consuming characteristics of private negotiations make the swap market less efficient and impede its development. From the perspective of international experiences, investors, as a rule, need to sign a uniform and standard master agreement, which defines such items as default identification, how to deal with default, and credit guarantee so as to reduce credit risk and cut down transaction cost. As a matter of fact, the above-mentioned uniform master agreement is about the formulation of a swap agreement. However, most of the literature materials concerning credit arbitrage of swaps are only limited to the equal allocation of profit and concrete analysis of the calculation content of swaps instead of generalizing the formulation of a swap agreement such as forming uniform calculation flow and establishing a model, which makes it difficult to make the agreement.

Thus, this paper focuses on how to establish a model as to the formulation of a swap agreement and make it easy to operate and seek solutions through equations, which will provide convenience for swap transactions.

### 2 Credit Arbitrage Model of Swaps

Although swaps don't have a long history, its new varieties have emerged one after another, among which the most important are interest rate swaps and currency swaps. Interest rate swaps refer to an agreement in which both parties agree to exchange cash flows based on the same amount of nominal principal in the same currency within a specific time limit in the future. One party's cash flow is calculated on the basis of a floating interest rate and the other party's cash flow on the basis of a fixed interest rate. Currency swaps refer to the exchange between the principal and fixed interest in one currency and the equivalent principal and fixed interest in another currency. The main reason for currency swaps lies in the comparative advantages of both parties in their own financial market. Since interest rate swaps involve no exchange of nominal principal but that of interests, the cash flows are

relatively simple. However, currency swaps involve both, so the model begins with interest rate swaps.

**2.1 Introduction of interest rate swap problem**

Assume that both Company A and Company B want to borrow ¥ M for n years. A wants to borrow at a k-month floating interest rate (measured by LIBOR), while B at a fixed rate. However, because their credit ratings are different, the interest rates that the market provides for them are different. See Table 1.

**Table 1 Borrowing Interest Rates the Market Provides for Companies A&B**

	IR	Fixed Rate	Floating Rate
Company A		$r_{A1}$	$r_{A2}$
Company B		$r_{B1}$	$r_{B2}$

(IR stands for interest rate)

In the table above, r represents annual compound interest. Let's assume that Company A has an absolute advantage, i.e., the borrowing interest rate for Company A is lower than that for Company B, no matter fixed rates or floating rates are involved, then  $r_{B1} > r_{A1}, r_{B2} > r_{A2}$ .

According to the Theory of Comparative Advantage proposed by David Ricardo, a famous economist in the United Kingdom of Great Britain and Northern Island, the following condition has to be met in order to be in conformity with the condition of swaps:  $r_{B1} - r_{A1} > r_{B2} - r_{A2}$ .

That is to say, Company A has a comparative advantage in the fixed rate. However, based on its actual needs, Company A only wants to borrow at a k-month floating rate; Company B has a comparative advantage in the floating rate (less disadvantaged), but based on its realities, Company B only wants to borrow at a fixed interest rate.

**2.2 Problem analysis**

According to the Theory of Comparative Advantage, both parties only have to borrow at a rate in which they have a comparative advantage. The key problem lies in the interest swap after borrowing, which determines the allocation of profit between both parties. Therefore, the model is to seek the total profit and then allocate it. Since the profit is the result of both parties' cooperation, it should be shared according to each party's contribution. Because they have to cooperate with each other in the swap, one method is to distribute the benefit equally and the other according to their different credit ratings.

**2.3 Model assumptions**

No market friction.

There exist a great number of ordinary companies that need swaps in the market; however, the number of companies with a high reputation is relatively small.

The floating interest rate is measured by LIBOR.

In general, k—the duration of the floating interest rate— is 3 or 6 months.

No inflation.

**2.4 Model establishment and solution seeking**

In order to work out the total profit, the different situations when there is a swap and no swap should be contrasted. And the cost reduced is the total profit.

Step 1: Calculate the original interest rates and the nominal borrowing interest rates.

The original interest rate for Company A  $r_{A0} = r_{A2}$ , the original interest rate for Company B  $r_{B0} = r_{B1}$ . According to the Theory of Comparative Advantage, costs can be reduced through a swap. The nominal borrowing program is as follows:

Company A borrows at a fixed rate of  $r_{A1}$ , then the nominal borrowing rate for Company A is  $r_{A1}$ . Company B borrows at a floating rate of  $r_{B2}$ , then the nominal borrowing rate for Company B is  $r_{B2}$ .

Step 2: calculate both parties' total yield r. The yield is the cost reduced, then the total yield is equal to the original interest rates minus the present nominal interest rates:  $r = r_{B1} - r_{A1} + r_{A2} - r_{B2}$

The profit is the result of both party's cooperation, so it should be shared. The share proportion should be determined by negotiation. And the simplest method to distribute the profit is by equal

allocation, that is, both parties' financing costs are reduced by  $\frac{1}{2}r$ . Suppose that  $p_A$  and  $p_B$  are respectively taken as the profit proportions for Companies A and B, then  $p_A = p_B = \frac{1}{2}r$ .

The other method is to allocate the profit according to both parties' advantageous position. The so called advantageous position is correlated with a company's credit rating. That is, the higher the credit rating is, the less risky the bank thinks of the lending is and less worried about the company's ability to repay its debt. Thus the lending interest that a bank provides for such a company is lower, which means lower borrowing cost for the company. Vice versa. Thus, a company's advantageous position is positively correlated with its credit rating, but negatively correlated with its borrowing interest rate.

A company with an advantageous position in the market can make use of its advantages so as to get a larger share of the profit. This is because from the perspective of supply and demand the number of companies with a high credit rating is relatively small while there are a lot of companies with a comparatively low credit rating. If a company with a relatively low credit rating does not accept the allocation method offered by a company with a high credit rating, it is rather difficult for such a company to find another company with a high credit rating.

Since the market has provided the data concerning interest rates and advantageous position of a company is negatively correlated with its borrowing interest rate, this paper adopts the inverse of interest rate to measure advantageous position. Therefore, the advantage function is defined as:

$d = \frac{1}{r_{fix}}$  ( $r_{fix}$  stands for a fixed borrowing interest rate). If each company's profit is allocated according to the value of advantage function, then

$$p_A = \frac{d_A}{d_A + d_B}, p_B = \frac{d_B}{d_A + d_B}, p_A + p_B = 1.$$

$p_A$  stands for Company A's profit ratio, while  $p_B$  stands for Company B's profit ratio. Therefore, the real interest rate at which Company A borrows  $r_A = r_{A2} - p_A \times r$ ; while the real interest rate at which Company B borrows  $r_B = r_{B1} - p_B \times r$ . This kind of allocation means lower cost for the company with a higher credit rating, but higher for the company with a lower credit rating.

Step 3: Work out the cash flows that should be exchanged.

After completing the calculation of the real and nominal interest rates at which both parties borrow, it's time to work out the cash flows that should be exchanged. Assume that Company A pays an interest rate of  $x$  to Company B, while Company B pays an interest rate of  $y$  to Company A. See Figure 1.

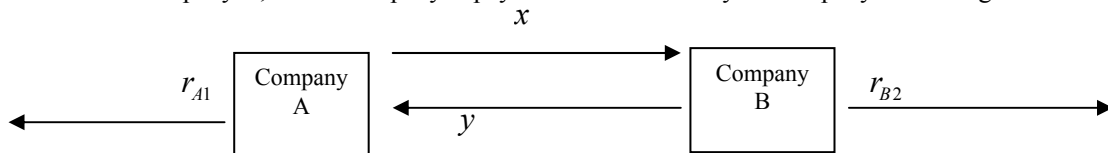


Figure 1 Flow of Arbitrage with Interest Rate Swaps

According to the balance of cash flow, i.e., the cost of Company A or Company B should be equal to the net cash inflow or outflow.

Company A:  $r_A = r_{A1} + x - y$ ; Company B:  $r_B = r_{B2} - x + y$ .

The two equations above are equivalent, that is

$$x - y = r_A - r_{A1} = r_{B2} - r_B = \frac{1}{2}((r_{A2} - r_{A1}) + (r_{B2} - r_{B1})) \tag{1}$$

$x$  on the left side of Formula 1 is the interest rate that Company A should pay to Company B and  $y$  the interest rate that Company B should pay to Company A. Besides, the principals involved in the interest rate swap are in the same kind of currency. Therefore, the left side of Formula 1 is the net cash flow when two companies settle accounts and Formula 1 is called settlement equation.

$(r_{A2} - r_{A1}) + (r_{B2} - r_{B1}) = r_{A2} + r_{B2} - r_{A1} - r_{B1}$  on the right side of Formula 1 can also be considered as subtraction of the total fixed interest rate from the total floating interest rate. If the floating interest rates in the market have increased a lot and the floating interest rates are much higher than the fixed interest

rates, that is,  $r_{A2} > r_{A1}, r_{B2} > r_{B1}$ , then  $x - y > 0$ , i.e., Company A should pay interest to Company B. Judged from the actual needs of Company A, Company A would have borrowed at a floating interest rate, thus the margin by which the floating interest rate has increased should be covered by Company A. Similar conclusions can be drawn from the other situations, therefore we get a Theorem.

Theorem: When the floating rate is higher on the date of payment, the company that wanted to borrow money at the floating rate has to pay interest to the other company that wanted to borrow at a fixed interest rate; when the floating rate is lower, the latter company should pay interest to the former.

The number of solutions  $(x, y)$  of the settlement equation is countless, i.e. the corresponding cash flows that should be exchanged are in great varieties. Thus, the selection of solutions should comply with usual usage and the formula above. The typical solution selection is as follows:

$$x = r_A, y = r_{A1}.$$

Then the flow of arbitrage with swaps is as follows:

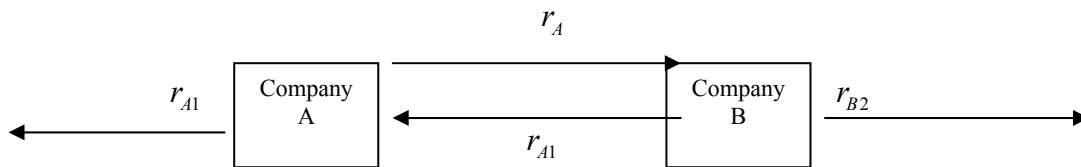


Figure 2 Flow of Arbitrage with Interest Rate Swaps

With the flow chart above, it is convenient to work out the interest that should be paid on the date of payment. In interest rate swaps, the interest should be paid every k months. Assume  $r_A > r_{A1}$ , then Company A shall pay Company B  $M \cdot p \cdot (r_A - r_{A1})$ . Otherwise, Company B shall pay company A  $M \cdot p \cdot (r_{A1} - r_A)$ .

The model above is embodied in simple calculation with clear steps. Therefore it is suitable for seeking solutions through equations. The model above is called the Credit Arbitrage Model of Swaps or CAMS.

**2.5 Improvement of model**

When there is a middleman involved in the swap, he would charge for the service he provides. Assume the charge is  $t$ , then the flow of arbitrage with swaps changes as follows:

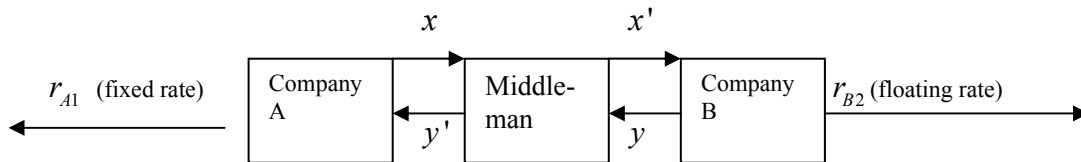


Figure 3 Flow of Arbitrage with Swaps Involving a Middleman

$x'$  and  $y'$  in the chart above represent the interest rate flows after deducting service fee charged by the middleman. The deduction of service fee leads to the decrease of the profit of Company A and Company B,  $r_A = r_{A2} - p_A \times (r - t), r_B = r_{B1} - p_B \times (r - t)$ . The middleman deducts the service charge from  $x$  and  $y$ . But as to how the service fee is deducted, Companies A and B have no interest in it. These two companies are only concerned about their cost. Therefore, we can let  $x = x', y = y' + t$  and then get the flow chart of swaps by making use of the settlement equation.

**3 Case Study**

Assume that Company A and Company B intend to borrow 5-year 10 million US dollars. Company A wants to borrow at a 6-month floating interest rate, while Company B wants to borrow at a fixed interest rate. But due to the different credit ratings of two companies, the market provides different lending interest rates to them. See Table 2. The interest rates in the table below refer to annual compound interest rate.

**Table 2 Lending Interest Rates the Market Provides for Companies A & B**

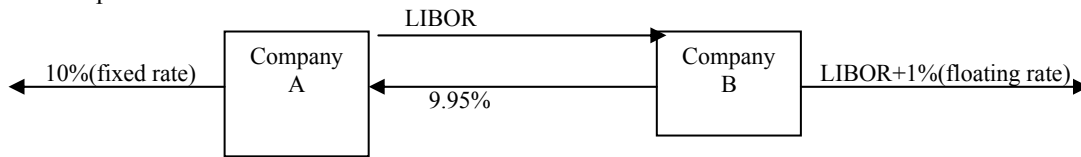
	Fixed Rate	Floating Rate
Company A	10.00%	6-month LIBOR+0.30%
Company B	11.20%	6-month LIBOR+1.00%

According to CAMS, it is calculated as follows:

The original interest rate for Company A  $r_{A0} = LIBOR + 0.30\%$ , The original interest rate for Company B  $r_{B0} = 11.2\%$ . Company A borrows at the fixed interest rate of  $r_{A1}$ , the nominal borrowing interest rate for Company A is 10%; Company B borrows at the floating interest rate of  $LIBOR + r_{B2}$ . The nominal interest rate for Company B is  $LIBOR + 1\%$ . Then

$$r = r_{B1} - r_{A1} + r_{A2} - r_{B2} = 0.5\% .$$

The real borrowing interest rate for Company A:  $r_A = r_{A0} - p_A r = LIBOR + 0.035\%$ ; The real borrowing interest rate for Company B  $r_B = r_{B0} - 0.235\% = 10.965\%$ . In contrast to equal allocation, Company A which has an absolute advantage, has saved more. Then the flow of arbitrage with interest rate swaps is as follows:



**Figure 4 Flow of Arbitrage with Interest Rate Swaps**

In the swap above, the interest should be paid every six months. Therefore it should be stipulated in the terms of swap agreement that one party should pay the other party the balance between the fixed rate and the floating rate. Assume that LIBOR is 11.00% on a certain date of payment. Then Company A should pay Company B 52, 500 US dollars, that is,  $1,000,000 \times 0.5 \times (11.00\% - 9.95\%)$ .

From the solution seeking process of the case above, it is discovered that it is easy to get the flow chart of interest rate swaps and the interest that should be paid from the Credit Arbitrage Model of Swaps. Therefore, the model mentioned above is very pragmatic.

#### 4 Conclusion

Financial swaps used to be well-known as the most important financial innovation in the western financial world since 1980s. With the boom of swap market, lack of uniform master agreement will impede the further development of China's financial derivative market and the exchange of cash flows also needs immediate standardization. The CAMS in this paper first designed the key profit allocating program successfully and then provided the detailed transaction of different companies. Case Study has demonstrated the operability and practicability of the model. In addition, this model needs further improvement. For example, this model should be extended to currency swaps.

#### References

- [1] Zhang Yichun, Zheng Zhenlong. Financial Market[M]. Beijing: Higher Education Press, 2003: 136-142 (In Chinese)
- [2] Huang Weiping. International Economics[M]. Beijing: University of International Business and Economics Press, 2008: 253-261 (In Chinese)
- [3] Zheng Zhenlong, Chen Rong. Financial Engineering[M]. Beijing: Higher Education Press, 2008:104-116 (In Chinese)
- [4] Zvi Bodie, Robert C.Merton. Finance[M]. Beijing: People's Publishing House of China, 2000: 278-279 (In Chinese)